INTERPRETATION OF STIFFNESS & DAMPING

Introduction

This sheet gives some insight about stiffness and damping and their effect on the dynamics of mechanical systems.

Influence of stiffness

Stiffness increases the tracking behavior the displacement x of the end-effector (mass m) in relation to the input x_{in} (a stiff actuator). Moreover, it decreases the influence of the external force F_e, which is often a disturbance to the system.

Influence of damping

Damping is difficult! Damping can be regarded as loss of energy. However, the positive effect of damping is that it damps oscillations and resonances.

Damping prediction

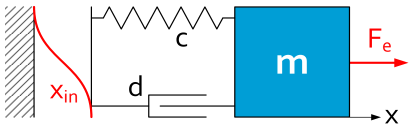
The damping of mechanical systems is hard to predict. Rule of thumb: damping decreases with increasing frequency. Joints and other system impurities increase damping.

$d = 2\zeta\sqrt{cm}$ with viscous damping ratio ζ:

System	ζ [-]
Metals in elastic range	0.01
Continuous metal structures	0.02 - 0.04
Metal structures with joints	0.03 - 0.07
Plastics (hard - soft)	0.02 - 0.05
Rubber	0.05
Sintered material (piezos)	0.05
Airpots (vibration isolation tables)	0.07

Schematic overview

1 mass m, 1 spring c, 1 damper d, input x_{in}, external force F_e



Differential equation

$m\ddot{x} + d(\dot{x} - \dot{x}_{in}) + c(x - x_{in}) = F_e$

Tracking - design rule

When designing a system that has to track the input x_{in} and that needs to be insensitive to disturbance force F_e, then design **'light and stiff'**.

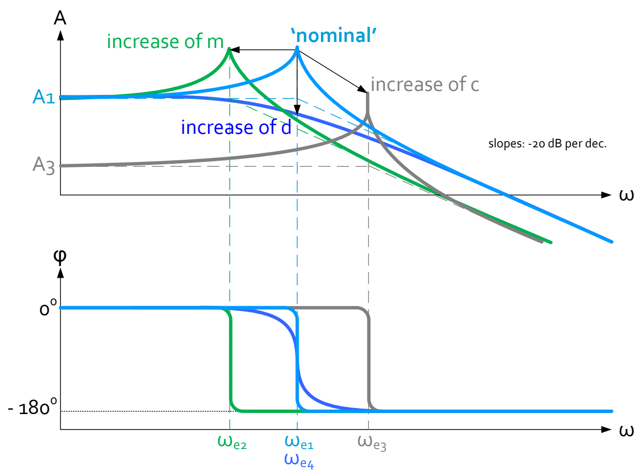
Vibration isolation - design rule

When designing a system that needs to be insensitive to vibrations x_{in} (such as ground vibrations), then design **'heavy and soft'**.

Eigen frequency

At this point the spring energy is converted into kinetic energy: $cx = m\ddot{x}$ hence: $c\hat{x} = m\hat{x}\omega^2$ and thus: $\omega = \sqrt{c/m}$

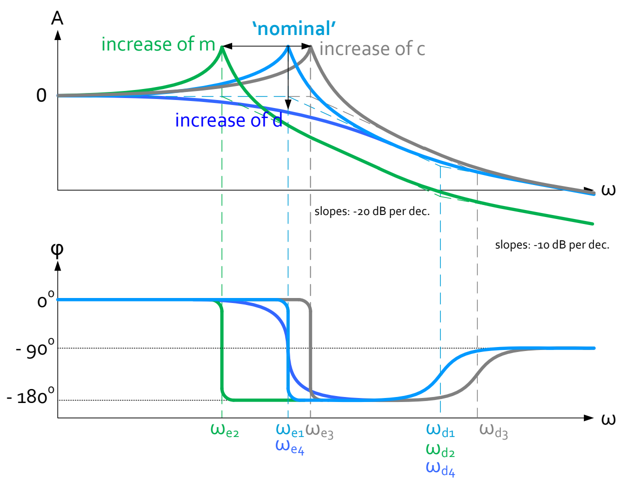
Response to the external force: $H_1 = \frac{x}{F_e} = \frac{1}{ms^2+ds+c}$



Properties:

- Eigen frequency: $\omega_{ei} = \sqrt{\frac{c}{m}} \sim \sqrt{c} \sim \frac{1}{\sqrt{m}}$
- Gain at $\omega = 0:\ A_i = \frac{1}{c} \sim \frac{1}{c}$

Response to the input: $H_1 = \frac{x}{x_{in}} = \frac{ds+c}{ms^2+ds+c}$



Properties:

- Eigen frequency: $\omega_{ei} = \sqrt{\frac{c}{m}} \sim \sqrt{c} \sim \frac{1}{\sqrt{m}}$
- 2^{nd} cross-over frequency: $\omega_{ei} = \frac{c}{d} \sim c \sim \frac{1}{d}$
  (from $cx = d\dot{x}$ hence: $c\hat{x} = d\hat{x}\omega$ and thus: $\omega = c/d$)
- Gain at $\omega = 0:\ A_i = \frac{c}{c} = 1 = 0\ dB$