Introduction

This sheet is to do a quick scan to the resonances of a desired transfer of a dynamic system via state space approach. From state space a bode-diagram can be created with appropriate software.

Equations of Motion (n-dimensional)

$\underline{M}\,\underline{\ddot{x}} + \underline{D}\,\underline{\dot{x}} + \underline{K}\,\underline{x} + \underline{F} = \underline{0}$

with $\underline{x} = [x_1 \quad \cdots \quad x_n]^T \rightarrow n\;x\;1$

$$\underline{M} = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & m_n \end{bmatrix} \rightarrow n\;x\;n$$

$$\underline{K} = \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ K_{n1} & K_{n2} & \cdots & K_{nn} \end{bmatrix} \rightarrow n\;x\;n$$

$K_{i,j} \rightarrow K_{mi,xj}$ = Sum of all c that work on m_i if x_j is moved*

$$\underline{D} = \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1n} \\ D_{21} & D_{22} & \cdots & D_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ D_{n1} & D_{n2} & \cdots & D_{nn} \end{bmatrix} \rightarrow n\;x\;n$$

$D_{i,j} \rightarrow D_{mi,xj}$ = Sum of all d that work on m_i if x_j is moved*

** Check for $\underline{K}$ and $\underline{D}$: If all elements of row or column I are summed the result is the stiffness or damping of mass I in relation to the fixed world (see example). Furthermore these matrices are*

$\underline{F}$ = external force $\rightarrow n\;x\;1$

(not composed of stiffness/dampers)

State Space form

(SISO, n-dimensional, time independent)

$$\underline{\dot{q}} = \underline{A}\underline{q} + \underline{B}u$$
$$y = \underline{C}\underline{q} + \underline{D}u$$

state vector: $\underline{q} = [x_1 \quad \cdots \quad x_n \;\; \dot{x}_1 \quad \cdots \quad \dot{x}_n]^T \rightarrow 2n\;x\;1$

u = input $\rightarrow 1\;x\;1$

u should be at least $\frac{d}{dt}$, so no x_i, always a flux or a multiplication of fluxes with parameters (see example).

y = output$\rightarrow 1\;x\;1$

y should be in the form x_i or $\dot{x}_i$ and multiplications with parameters are possible but no double flux or higher fluxes (see example).

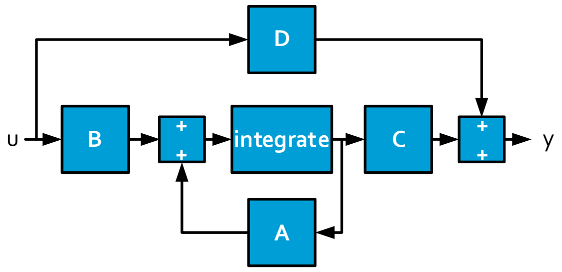
$$\underline{A} = \begin{bmatrix} \underline{0}_{nxn} & \underline{I}_{nxn} \\ \underline{M}^{-1}\underline{K} & \underline{M}^{-1}\underline{D} \end{bmatrix} \rightarrow 2n\;x\;2n \text{ (system matrix)}$$

$\underline{B} \rightarrow 2n\;x\;1$ is the input matrix; composition see examples

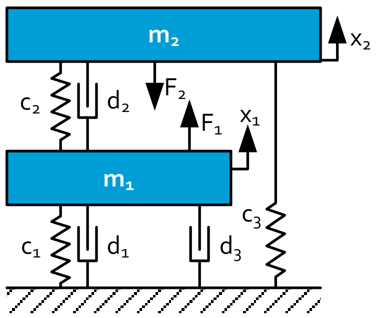
$\underline{C} \rightarrow 2n\;x\;1$ is the output matrix; composition see examples

This document assumes no feed forward so: $\underline{D} = \underline{0}$

Block diagram representation



Example



$\underline{x} = [x_1 \quad x_2]^T$ so: $n = 2$

$\underline{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$ with $m_0 = 0$

$\underline{K} = \begin{bmatrix} -c_1 - c_2 & c_2 \\ c_2 & -c_2 - c_3 \end{bmatrix}$

$\underline{D} = \begin{bmatrix} -d_1 - d_2 - d_3 & d_2 \\ d_2 & -d_2 \end{bmatrix}$

$\underline{F} = [\,F_1 \quad -F_2]$

$\underline{q} = [x_1 \quad x_2 \quad \dot{x}_1 \quad \dot{x}_2]^T$

$\underline{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \multicolumn{2}{c}{\underline{M}^{-1}\underline{K}} & \multicolumn{2}{c}{\underline{M}^{-1}\underline{D}} \end{bmatrix}$ with $M^{-1} = \begin{bmatrix} 1/m_1 & 0 \\ 0 & 1/\,m_2 \end{bmatrix}$

3 examples of inputs:

$u_1 = \ddot{x}_1\,,\; u_2 = F_1\,,\; u_3 = \dot{x}_2 - \dot{x}_1$

Thus;

$\underline{B}_1 = [0 \quad 0 \quad 1 \quad 0]^T$

$\underline{B}_2 = \begin{bmatrix} 0 & 0 & \frac{1}{m_1} & 0 \end{bmatrix}^T$

$\underline{B}_3 = [-1 \quad 1 \quad 0 \quad 0]^T$

3 examples of outputs:

$y_1 = \dot{x}_2\,, y_2 = x_2 - x_1\,, y_3 = c_2(x_2 - x_1) + d_2(\dot{x}_2 - \dot{x}_1)$

Thus;

$\underline{C}_1 = [0 \quad 0 \quad 0 \quad 1]^T$

$\underline{C}_2 = [-1 \quad 1 \quad 0 \quad 0]^T$

$\underline{C}_3 = [-c_2 \quad c_2 \quad -d_2 \quad d_2]^T$

$\underline{D} = \underline{0}$