**Introduction**

When a straight beam is subjected to an axial moment, each cross section twists around its torsional center. Shear stresses occur within the cross sectional planes of the beam.

**Angular twist**

For a torsionally loaded beam, the angular twist is described by:

\[
\varphi = \frac{T \cdot l}{G \cdot J_T}
\]

\(G\) is the shear modulus. The relation between the shear modulus \(G\) and the elastic modulus \(E\) is defined by the following formula:

\[
G = \frac{E}{2(1+v)} \approx 0.38E \quad \text{ (For most metals)}
\]

**Rotational stiffness**

The rotational stiffness of a torsionally loaded beam is:

\[
K_T = \frac{T}{\varphi} = \frac{G \cdot J_T}{l}
\]

**Maximum torque load**

For a torsionally loaded beam, the maximum torque load can be calculated with:

\[
T_{\text{max}} = \frac{J_T \tau_{\text{max}}}{y}
\]

\(J_T\) is the torsion constant. It is equal to the polar moment of inertia \(I_z\) if the cross section is circular.

For non-circular cross sections warping occurs which reduces the effective torsion constant. For these shapes, approximate solutions of the torsion constant are given in the table below.

**Torsion constant**

<table>
<thead>
<tr>
<th>Cross section</th>
<th>Torsion constant (J_T)</th>
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</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Circular Cross Section" /></td>
<td>(J_T = I_z = \frac{\pi}{2} r^4)</td>
<td><img src="image2.png" alt="Circular Cross Section" /></td>
<td>(J_T = I_z = \frac{\pi}{2} (r_o^4 - r_i^4))</td>
</tr>
<tr>
<td><img src="image3.png" alt="Rectangular Cross Section" /></td>
<td>(J_T \approx \frac{9}{64} w^4)</td>
<td><img src="image4.png" alt="Rectangular Cross Section" /></td>
<td>(J_T \approx t(w - t)^3)</td>
</tr>
<tr>
<td><img src="image5.png" alt="Rectangular Cross Section" /></td>
<td>With (h\geq w) (J_T \approx hw^3 \left(1 - 0.21 \frac{w}{h} \left(1 - \frac{w}{12h}\right)\right))</td>
<td><img src="image6.png" alt="Rectangular Cross Section" /></td>
<td>(J_T \approx \frac{2t(h - t)^2(w - t)^2}{h + w - 2t})</td>
</tr>
</tbody>
</table>


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