**Introduction**

This sheet gives some insight about stiffness and damping and their effect on the dynamics of mechanical systems.

**Influence of stiffness**

Stiffness increases the tracking behavior of the displacement $x$ of the end-effector (mass $m$) in relation to the input $x_{in}$ (a stiff actuator). Moreover, it decreases the influence of the external force $F_e$, which is often a disturbance to the system.

**Influence of damping**

*Damping is difficult!* Damping can be regarded as loss of energy. However, the positive effect of damping is that it damps oscillations and resonances.

**Damping prediction**

The damping of mechanical systems is hard to predict. Rule of thumb: damping decreases with increasing frequency. Joints and other system impurities increase damping. $d = 2 \varepsilon \sqrt{cm}$ with viscous damping ratio $\varepsilon$:

<table>
<thead>
<tr>
<th>System</th>
<th>$\varepsilon$ [-]</th>
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</thead>
<tbody>
<tr>
<td>Metals in elastic range</td>
<td>0.01</td>
</tr>
<tr>
<td>Continuous metal structures</td>
<td>0.02 - 0.04</td>
</tr>
<tr>
<td>Metal structures with joints</td>
<td>0.03 - 0.07</td>
</tr>
<tr>
<td>Plastics (hard - soft)</td>
<td>0.02 - 0.05</td>
</tr>
<tr>
<td>Rubber</td>
<td>0.05</td>
</tr>
<tr>
<td>Sintered material (piezos)</td>
<td>0.05</td>
</tr>
<tr>
<td>Airpots (vibration isolation tables)</td>
<td>0.07</td>
</tr>
</tbody>
</table>

**Response to the external force:**

$H_1 = \frac{x}{F_e} = \frac{1}{ms^2 + ds + c}$

**Response to the input:**

$H_1 = \frac{x}{x_{in}} = \frac{ds + c}{ms^2 + ds + c}$

**Schematic overview**

1 mass $m$, 1 spring $c$, 1 damper $d$, input $x_{in}$, external force $F_e$.

**Differential equation**

$m \ddot{x} + d(\dot{x} - \dot{x}_{in}) + c(x - x_{in}) = F_e$

**Tracking - design rule**

When designing a system that has to track the input $x_{in}$ and that needs to be insensitive to disturbance force $F_e$, then design ‘light and stiff’.

**Vibration isolation - design rule**

When designing a system that needs to be insensitive to vibrations $x_{in}$ (such as ground vibrations), then design ‘heavy and weak’.

**Eigen frequency**

At this point the spring energy is converted into kinetic energy: $c \ddot{x} = m \dot{x}^2$ hence: $c \dot{x} = m \dot{x} \omega^2$ and thus: $\omega = \sqrt{c/m}$

**Response to the input:**

$H_1 = \frac{x}{x_{in}} = \frac{ds + c}{ms^2 + ds + c}$

**Properties:**

- Eigen frequency: $\omega_{el} = \frac{c}{\sqrt{m}} \approx \sqrt{c} \approx \frac{1}{\sqrt{m}}$
- Gain at $\omega = 0$: $A_i = \frac{c}{d} \approx \frac{1}{c}$
- $2^{nd}$ cross-over frequency: $\omega_{el} = \frac{c}{d} \approx c \approx \frac{1}{d}$

**Properties:**

- Eigen frequency: $\omega_{el} = \frac{c}{\sqrt{m}} \approx \sqrt{c} \approx \frac{1}{\sqrt{m}}$
- Gain at $\omega = 0$: $A_i = \frac{c}{d} = 1 = 0 \text{ dB}$